Scattering theory and $1/N$ expansion in the chiral Gross-Neveu model

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We show how to establish the scattering theory of the chiral Gross-Neveu model in terms of a massive field with generalized statistics. It is in terms of this field that the property of antiparticles being bound states of particles is consistently formulated. We also employ an auxiliary local Fermi field in order to develop an infrared-regular $1/N$ expansion. The connection between these two fields is discussed.

I. INTRODUCTION

Recently, considerable interest has been devoted to obtaining the exact S matrix of all two-dimensional factorizable field-theoretical models.[1] Among these the chiral Gross-Neveu model[2] defined by the Lagrangian

$$\mathcal{L}=i\sum_{i=1}^{N}\bar{\psi}_i\partial\!\!\!/\psi_i+\tfrac{1}{2}g^2\left[\left(\sum_{i=1}^{N}\bar{\psi}_i\psi_i\right)^2-\left(\sum_{i=1}^{N}\bar{\psi}_i\gamma^5\psi_i\right)^2\right], \quad (1)$$

although suspected for a long time of belonging to this class, was plagued by difficulties which delayed its complete comprehensibility. These were related to an apparent spontaneous breakdown of chiral invariance, which is of course impossible in two dimensions.[3]

However, it has been argued by Witten[4] that dynamical mass generation for the fermions can be reconciled with the absence of spontaneous symmetry breaking. As a consequence, several proposals for the exact S matrix of this model have been made.[5-7] In Ref. 5 a unique S matrix was proposed following the guiding principle that in this model antiparticles are bound states of particles. In Ref. 7 it was pointed out that uniqueness of the above S matrix can also be achieved by demanding absence of ghosts.[8]

In this paper we want to elaborate on the scattering theory of the chiral Gross-Neveu model. Since the natural interpolating field $\hat{\psi}$ has generalized statistics,[5] we find it convenient in Sec. II to introduce an auxiliary field ψ', obeying Fermi statistics, in terms of which conventional scattering theory can be formulated. The interpolating field $\hat{\psi}$ satisfies a generalized asymptotic condition which allows us to relate its S matrix to that of ψ'. A certain order of the particles in the in and out states is required, if the S matrix of the $\hat{\psi}$ and ψ' fields is to coincide. In Sec. III these points are illustrated in detail for SU(2) and SU(3).

The field ψ', being canonical, allows one to check the proposed exact S matrix in perturbation theory. In Sec. IV we work out the $1/N$ expansion for the Green's functions and find out, as expected from boson-representation arguments presented in Sec. II, that they belong to a genuine massive theory, i.e., they are free from any infraparticle structure. On the mass shell our results coincide with those of Ref. 7, reproducing to first order in $1/N$ the proposed exact scattering amplitudes. Section V is reserved for a brief conclusion.

II. THE FIELDS OF THE CHIRAL GROSS-NEVEU MODEL

The original field ψ of the chiral Gross-Neveu model can be written in boson form as[9]

$$\psi_i(x)=K_i\left(\frac{m}{2\pi}\right)^{1/2}e^{(\pi/4)\gamma^5}\exp\left\{-i\left(\frac{\pi}{N}\right)^{1/2}\left[\gamma^5\phi(x)+\int_x^\infty dz^1\dot{\phi}(z)\right]\right.$$
$$\left.-i\sqrt{\pi}\left[\gamma^5\phi_i(x)+\int_x^\infty dz^1\dot{\phi}_i(z)\right]\right\},\quad i=1,2,\ldots,N \quad (2a)$$

$$\sum_{i=1}^{N}\phi_i(x)=0\,, \quad (2b)$$

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where K_i is a Klein factor needed to ensure the anticommutation relations for the different SU(N) components, $\phi(x)$ is the pseudopotential of the U(1) current, and the $\phi_i(x)$ are pseudopotentials of the diagonal $SU(N)$ currents. $\phi(x)$ is a free canonical zero-mass field, as a consequence of the U(1) × U(1) symmetry, and this implies that $\psi_i(x)$ describes infraparticles.[10]

In order to extract the real particle content of the theory, we introduce the fields $\hat{\psi}$ defined as

$$\hat{\psi}_i(x)=K_i\left(\frac{m}{2\pi}\right)^{1/2}e^{i(\pi/4)\gamma^5}\exp\left\{-i\sqrt{\pi}\left[\gamma^5\phi_i(x)+\int_x^\infty dz^1\dot{\phi}_i(z)\right]\right\},\quad i=1,2,\ldots,N. \tag{3}$$

From its definition it is clear that $\hat{\psi}$ carries neither U(1) chirality nor the U(1) charge and consequently transforms according to SU(N). It is therefore not surprising that the $\hat{\psi}_i$ satisfy the algebraic identity

$$\hat{\psi}_i^*=\frac{1}{(N-1)!}\left(\prod_{i=1}^{N}K_i\right)\epsilon_{ij_1j_2\cdots j_{N-1}}\hat{\psi}_{j_1}\cdots\hat{\psi}_{j_{N-1}}, \tag{4}$$

where a suitable normal-product prescription for the right-hand side is understood. In a theory with particle content Eq. (4) means that antiparticles should be identified with bound states of $N-1$ particles.

We may, furthermore, read from definition (3) that $\hat{\psi}$ satisfies neither Fermi nor Bose statistics, but rather carries "spin" $S=\frac{1}{2}(1-1/N)$. Therefore one expects an appropriate collision limit to reveal asymptotic states characterized by generalized statistics.[9]

Since a Lehmann-Symanzik-Zimmermann formalism appropriate to the field $\hat{\psi}$ is not known to us, we are going to introduce an auxiliary Fermi field ψ',[11] in terms of which the usual scattering theory can be carried through,

$$\psi_i'(x)=\exp\left\{i\left(\frac{\pi}{N}\right)^{1/2}[\gamma^5A(x)+B(x)]\right\}\psi_i(x),\qquad i=1,2,\ldots,N \tag{5}$$

where $A(x)$ and $B(x)$ are two independent auxiliary, free massless fields, quantized with metric opposite to that of $\phi(x)$. The role of these fields is to compensate the infrared structure of ψ without affecting its statistics. In this way the U(1) × U(1) charges become spurions by a mechanism analogous to the one occurring in the massless Schwinger model,[12] the essential difference being that here the spurions carry nontrivial spin. It is thus clear that the Green's functions of ψ' will have the structure of a conventional massive, local fermionic theory. Consequently, following Haag and Araki,[13] we have

$$\psi'(vt,t)\xrightarrow[t\to+\infty(-\infty)]{}\frac{1}{\sqrt{|t|}}[e^{-im\gamma t}a_{\text{out(in)}}(m\gamma v)+e^{im\gamma t}b^\dagger_{\text{out(in)}}(m\gamma v)], \tag{6a}$$

where a and $b^\dagger$ are the usual annihilation and creation operators and $\gamma=(1-v^2)^{-1/2}$.

For $\hat{\psi}$ we expect a similar asymptotic limit

$$\hat{\psi}(vt,t)\xrightarrow[t\to+\infty(-\infty)]{}\frac{1}{\sqrt{|t|}}[e^{-im\gamma t}\hat{a}_{\text{out(in)}}(m\gamma v)+e^{im\gamma t}\hat{b}^\dagger_{\text{out(in)}}(m\gamma v)]. \tag{6b}$$

From the equal-time commutation relations for $\hat{\psi}$

$$\hat{\psi}_i(x,t)\hat{\psi}_i(y,t)=e^{2\pi iS\epsilon(x-y)}\hat{\psi}_i(y,t)\hat{\psi}_i(x,t), \tag{7}$$

we expect using the limit (6b) that

$$\hat{a}^\dagger_{\text{out(in)}}(p)\hat{a}^\dagger_{\text{out(in)}}(p')=e^{\pm 2\pi iS\epsilon(p-p')}\hat{a}^\dagger_{\text{out(in)}}(p')\hat{a}^\dagger_{\text{out(in)}}(p) \tag{8}$$

and the same relation with a replaced by b. A solution of Eq. (8) is given by

$$\hat{a}^\dagger_{\text{in}}(p)=a^\dagger_{\text{in}}(p)\exp\left[2\pi i(S-\tfrac{1}{2})\int_p^\infty N_{\text{in}}(p')dp'\right], \tag{9a}$$

where $N_{\text{in}}(p)$ is the total in-number density operator. From PT invariance we obtain the solution for the out operator

$$\hat{a}^\dagger_{\text{out}}(p)=a^\dagger_{\text{out}}(p)\exp\left[2\pi i(S-\tfrac{1}{2})\int_{-\infty}^p N_{\text{out}}(p')dp'\right]. \tag{9b}$$

Equations (9) generalize expressions obtained in the study of the scaling limit of the Ising model.[14]

We want to stress at this point that we consider $\hat{a}^\dagger$ and $\hat{b}^\dagger$ as our fundamental operators in terms of which the idea that antiparticles are bound states of particles can be consistently formulated. On the other hand, the whole factorizable, analytic S-matrix program is to be carried out using ψ' as an interpolating field. Notice though that from Eq. (9) the following identity holds:

$$\langle 0|a_{\text{out}}(p_1')\cdots a_{\text{out}}(p_n')a^\dagger_{\text{in}}(p_1)\cdots a^\dagger_{\text{in}}(p_n)|0\rangle=\langle 0|\hat{a}_{\text{out}}(p_1')\cdots\hat{a}_{\text{out}}(p_n')\hat{a}^\dagger_{\text{in}}(p_1)\cdots\hat{a}^\dagger_{\text{in}}(p_n)|0\rangle, \tag{10}$$

where $p_1 > p_2 > \cdots > p_n$, $p_1' > p_2' > \cdots > p_n'$. Hence using this order for the operators, S-matrix elements calculated with ψ' coincide with the ones one would obtain using $\hat{\psi}$, whereas the conventional ordering introduces nonanalytic phase factors in Eq. (10). In the following section we will discuss how the above ideas may be used to determine uniquely the S matrix of the chiral Gross-Neveu model.

III. THE SCATTERING MATRIX OF THE CHIRAL GROSS-NEVEU MODEL

As has been discussed already in Sec. II the correct invariance group of the present model in SU(N) [as opposed to U(N)]. The scattering problem having this symmetry is set up using the field $\hat{\psi}$. Since this field carries no U(1) charge, antiparticles have been identified with bound states of $(N-1)$ particles.

On the other hand, using the field ψ' the same problem may be viewed as having an apparent U(N) symmetry and Fermi statistics. Equation (10) now requires the U(N)-symmetric S matrix to have a bound state in the $\{\overline{N}\}$ channel with the same mass as the original particle. Consistency further requires that the scattering amplitude of this bound state with the original particle coincides with the particle-antiparticle S-matrix element computed using $\hat{\psi}$ as an interpolating field. From the general classification[15] of all factorizable U(N)-symmetric S matrices the above requirement uniquely fixes the scattering amplitudes of the chiral Gross-Neveu model to be[5]

$$\langle\delta(\theta_2)\gamma(\theta_1)|S|\alpha(\theta_1)\beta(\theta_2)\rangle = u_1(\varphi)\delta_{\alpha\gamma}\delta_{\beta\delta} + u_2(\varphi)\delta_{\alpha\delta}\delta_{\beta\gamma}\,, \tag{11a}$$

$$\langle\overline{\delta}(\theta_2)\gamma(\theta_1)|S|\alpha(\theta_1)\overline{\beta}(\theta_2)\rangle = t_1(\varphi)\delta_{\alpha\gamma}\delta_{\beta\delta} + t_2(\varphi)\delta_{\alpha\beta}\delta_{\gamma\delta}\,, \tag{11b}$$

where $\varphi = (\theta_1 - \theta_2)/i\pi$, and

$$u_1(\varphi) = \frac{\Gamma(1-\varphi/2)\Gamma(\varphi/2 - 1/N)}{\Gamma(1-\varphi/2-1/N)\Gamma(\varphi/2)}\,, \quad u_2(\varphi) = -\frac{2}{N\varphi}u_1(\varphi)\,, \tag{12a}$$

$$t_1(\varphi) = \frac{\Gamma(1/2+\varphi/2)\Gamma(1/2-\varphi/2-1/N)}{\Gamma(1/2-\varphi/2)\Gamma(1/2+\varphi/2-1/N)}\,, \quad t_2(\varphi) = -\frac{2}{N(1-\varphi)}t_1(\varphi)\,. \tag{12b}$$

The above amplitudes, which have been constructed with the local field ψ', satisfy the usual analyticity and crossing properties. They belong to class II of the general classification of Ref. 15. From Eq. (12a) one sees that the antisymmetric amplitude $u_1 - u_2$ has a bound state with mass given by

$$m_2 = m\frac{\sin(2\pi/N)}{\sin(\pi/N)}\,, \tag{13}$$

inducing now[16] the following spectrum

$$m_n = m\frac{\sin(n\pi/N)}{\sin(\pi/N)}\,, \quad 1 \le n \le N-1\,. \tag{14}$$

This equation implies that the bound state of $(N-1)$ particles has mass $m_{N-1} = m$. The consistency check mentioned above will now be illustrated for the case $N=3$. Computing the scattering amplitude of three particles, with two of them projected onto the $\{\overline{3}\}$ channel, we obtain for the ψ' S matrix, the relation

$$\frac{1}{\sqrt{2}}\epsilon_{\Sigma'\alpha'\beta'}\frac{1}{\sqrt{2}}\epsilon_{\Sigma\alpha\beta}\ {}_{\text{in}}\langle\gamma'(\theta_3)\beta'(\theta_2)\alpha'(\theta_1)|S|\alpha(\theta_1)\beta(\theta_2)\gamma(\theta_3)\rangle_{\text{in}} = -(u_1 - u_2)(\delta_{\Sigma'\Sigma}\delta_{\gamma'\gamma}T_1 + \delta_{\Sigma'\gamma'}\delta_{\Sigma\gamma}T_2)\,, \tag{15a}$$

where

$$T_1 = \tfrac{1}{2}(-2u_1u_2 - u_1u_2 - u_2u_1 + u_2u_2)\,, \tag{15b}$$

$$T_2 = \tfrac{1}{2}(u_1u_2 + u_2u_1 - u_2u_2)\,. \tag{15c}$$

From the residue of the amplitude at the pole $\theta_{12} = \frac{2}{3}i\pi$ one finds the scattering amplitude of the bound state $\overline{\Sigma}$ with the particle γ to be

$${}_{\text{in}}\langle\gamma'(\theta_2)\overline{\Sigma}'(\theta_1)|S|\overline{\Sigma}(\theta_1)\gamma(\theta_2)\rangle_{\text{in}} = -(\delta_{\Sigma'\Sigma}\delta_{\gamma'\gamma}T_1 + \delta_{\Sigma'\gamma'}\delta_{\Sigma\gamma}T_2)\,. \tag{16}$$

Equation (10) allows us to obtain the S matrix for $\hat{\psi}$, denoted by $\hat{S}$, to be

$${}_{\text{in}}\langle\gamma'(\theta_2)\overline{\Sigma}'(\theta_1)|S|\overline{\Sigma}(\theta_1)\gamma(\theta_2)\rangle_{\text{in}} = {}_{\text{in}}\langle\overline{\Sigma}'(\theta_1)\gamma'(\theta_2)|S|\overline{\Sigma}(\theta_1)\gamma(\theta_2)\rangle_{\text{in}} = {}_{\text{in}}\langle\overline{\Sigma}'(\theta_1)\gamma'(\theta_2)|\hat{S}|\overline{\Sigma}(\theta_1)\gamma(\theta_2)\rangle_{\text{in}}\,. \tag{17}$$

From Eqs. (11b) and (10) we obtain for the particle-antiparticle amplitudes

$${}_{\text{in}}\langle\overline{\delta}'(\theta_1)\gamma'(\theta_2)|\hat{S}|\overline{\delta}(\theta_1)\gamma(\theta_2)\rangle_{\text{in}} = -(t_1\delta_{\delta\delta'}\delta_{\gamma\gamma'} + t_2\delta_{\delta\gamma}\delta_{\delta'\gamma'})\,. \tag{18}$$

From Eq. (4) we see that the last amplitude of Eq. (17) is to be identified with that of Eq. (18) up to a Klein factor. From its definition the Klein factor of Eq. (4) has to have eigenvalues ±1. At the same time Eq. (4) requires this factor to belong to the center of the group. Consequently, for all odd N the Klein factor equals one. For SU(3) this means that the following identifications have to be valid:

$$T_i = t_i\,, \quad i=1,2\,. \tag{19}$$

In fact the explicit computation for T_i, Eqs. (15) and (12), confirms Eq. (19).

Finally, let us illustrate the role played by the Klein factor in Eq. (4) in the case $N=2$. Although in this case Eq. (4) does not imply a bound state, it nevertheless can be used to make the necessary identifications. From

$$\hat{\psi}^{*}_{\alpha} = e^{2i\pi Q_3}\epsilon_{\alpha\beta}\hat{\psi}_{\beta}\,, \tag{20}$$

where Q_3 is the third component of isospin, one finds

$$\hat{b}^{\dagger}_{\alpha} = e^{2i\pi Q_3}\epsilon_{\alpha\beta}\hat{a}^{\dagger}_{\beta}\,. \tag{21}$$

From Eq. (21) we get

$$\begin{aligned} &{}_{\rm in}\langle\bar{\gamma}(\theta_1)\delta(\theta_2)|\hat{S}|\bar{\alpha}(\theta_1)\beta(\theta_2)\rangle_{\rm in} \\ &\quad = -\epsilon_{\gamma\gamma'}\epsilon_{\alpha\alpha'}\,{}_{\rm in}\langle\gamma'(\theta_1)\delta(\theta_2)|\hat{S}|\alpha'(\theta_1)\beta(\theta_2)\rangle_{\rm in}\,, \end{aligned} \tag{22}$$

where a minus sign arose from the Klein factor in conjunction with the Eq. (21) used above. Equation (22) together with Eq. (10) implies

$$t_1\delta_{\alpha\gamma}\delta_{\beta\delta} + t_2\delta_{\alpha\beta}\delta_{\delta\gamma} = -\epsilon_{\gamma\gamma'}\epsilon_{\alpha\alpha'}(u_1\delta_{\alpha'\gamma'}\delta_{\beta\delta} + u_2\delta_{\alpha'\delta}\delta_{\gamma'\beta})\,, \tag{23}$$

hence

$$t_1 = -(u_1+u_2)\,, \quad t_2 = u_2\,. \tag{24}$$

This equation is confirmed using the formulas (11) and (12) for $N=2$.

IV. THE 1/N EXPANSION

With the aid of the local Fermi field ψ' we work out the $1/N$ perturbative approach and show in lowest order that the Green's functions correspond to a genuine massive theory, leading to the proposed exact S-matrix elements. On the mass shell our results coincide with those of Ref. 7, who resort to a field ψ which is apparently nonlocal and in this case would not lead to the usual Feynman rules.

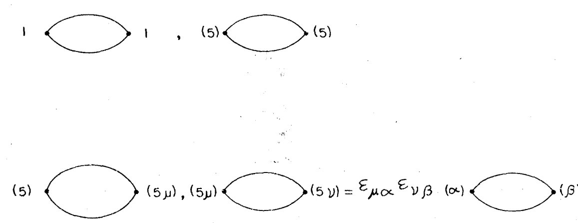

FIG. 1. Bubbles to be summed giving propagators to first order in $1/N$. ($\mu5$) stands for $\gamma^\mu\,\gamma^5$, etc.

From its definition Eq. (5) and Eq. (1) ψ' is described by the Lagrangian

$$\begin{aligned} \mathcal{L} = i\sum_{i=1}^{N}\bar{\psi}'_i\,\partial\!\!\!/\,\psi_i + \tfrac{1}{2}g^2\left[\left(\sum_{i=1}^{N}\bar{\psi}'_i\psi'_i\right)^2 - \left(\sum_{i=1}^{N}\bar{\psi}'\gamma^5\psi'\right)^2\right] \\ -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 + \bar{\psi}'\left(\frac{\alpha}{\sqrt{N}}\gamma^5\partial\!\!\!/ A - \frac{\beta}{\sqrt{N}}\partial\!\!\!/ B\right)\psi'\,, \end{aligned} \tag{25}$$

where the minus sign in the kinetic energy terms of A and B is due to their indefinite-metric quantization. The constants α and β are unrenormalized and, as we will show below, renormalization effects will have to be taken into account. Following standard procedures[2] the Green's functions in the $1/N$ expansion are generated by the following Feynman rules, having effective propagators given by

$$\tilde{\Delta}_\pi(P) = -\frac{2\pi i}{N}\,\frac{\coth(\varphi/2)}{\varphi}\,\frac{[1-\alpha^2/\pi\,(1-\varphi/\sinh\varphi)]}{1-\alpha^2/\pi}\,, \tag{26a}$$

$$\tilde{\Delta}_\sigma(P) = -\frac{2\pi i}{N}\,\frac{\tanh(\varphi/2)}{\varphi}\,, \tag{26b}$$

$$\tilde{\Delta}_A(P) = -\frac{i}{N}\,\frac{1}{P^2}\,\frac{1}{1-\alpha^2/\pi}\,, \tag{26c}$$

$$\tilde{\Delta}_B(P) = -\frac{i}{N}\,\frac{1}{P^2}\,, \tag{26d}$$

where $P^2 = -4m^2\sinh^2(\varphi/2)$. These propagators are obtained summing the bubbles of Fig. 1. The A propagator is the free one, as expected from Eq. (5), owing to Ward identities relating these bubbles, since for a free field of mass m one has

$$i\partial_\mu\langle T[\bar{\psi}\gamma^5\gamma^\mu\psi](x)[\bar{\psi}\gamma^5\gamma^\nu\psi](y)\rangle = 2m\langle T[\bar{\psi}\gamma^5\psi](x)[\bar{\psi}\gamma^5\gamma^\nu\psi](y)\rangle + \text{ST}\,, \tag{27a}$$

$$i\partial_\mu\langle T[\bar{\psi}\gamma^5\gamma^\mu\psi](x)[\bar{\psi}\gamma^5\psi](y)\rangle = 2m\langle T[\bar{\psi}\gamma^5\psi](x)[\bar{\psi}\gamma^5\psi](y)\rangle + i\langle\bar{\psi}\psi(x)\rangle\delta(x-y)\,, \tag{27b}$$

They imply the conservation of the true axial-vector current of ψ',

$$\partial_\mu\bar{\psi}'\gamma^\mu\gamma^5\psi' = \Box A = 0\,. \tag{28}$$

The B propagator remains free, since this field is a pure gauge excitation.

The $1/N$ correction to the four-point function is given by the graphs of Fig. 2. From our argu-

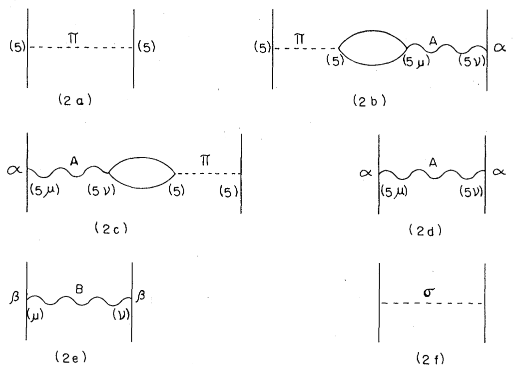

FIG. 2. Diagrams contributing to the four-point function to first order in $1/N$.

ments of Sec. II we expect that the choice of α and β corresponding to renormalized couplings given by $\sqrt{\pi}$ gives rise to the Green's functions free from infrared singularities. Indeed taking the limit $\alpha^2 \to \infty$ one notices that the graphs of Figs. 2(b) and 2(c) cancel on the mass shell and the pole in the pion propagator of Fig. 2(a) disappears. Its role is taken by the A propagator of Fig. 2(d).

The fact that the limit $\alpha^2 \to \infty$ implies a renormalized coupling given by $\sqrt{\pi}$ is heuristically evident, since the $(\gamma^\mu\gamma^5, \gamma^\nu\gamma^5)$ bubble has been regularized gauge invariantly. This produces a chiral charge $Q_5 = \int(\dot{\phi} + \dot{A})dx^1$, which commutes with the field ψ', so that in order to maintain a coupling of A with ψ' in the Lagrangian (25), the unrenormalized coupling must be boosted to infinity. This would not be the case had we used a different regularization for the $(\gamma^\mu\gamma^5, \gamma^\nu\gamma^5)$ bubble, for example one which preserves chiral invariance in the limit $m \to 0$. Of course these different regularizations are related by renormalization-group equations, ensuring that observable matrix elements are unchanged.

The four-point function computed from the graphs of Fig. 2 is given by

$$\Gamma_4 = \frac{-i\pi}{N}\gamma^\mu\gamma^5 \otimes \gamma^\nu\gamma^5 \frac{k_\mu k_\nu}{k^2} + \frac{i\beta^2}{N}\gamma^\mu \otimes \gamma^\nu \frac{k_\mu k_\nu}{k^2} + \gamma^5 \otimes \gamma^5 [\lim_{\alpha^2 \to \infty} \tilde{\Delta}_\pi(k)] + 1 \otimes 1 \tilde{\Delta}_\sigma(k) + \cdots, \tag{29}$$

where the dots stand for the graphs of Figs. 2(b) and 2(c).

Notice that for $\beta = \sqrt{\pi}$ the vector and pseudovector couplings combine to give a contact term, so that the massless excitations have in fact been removed. On the mass shell our Green's functions unambiguously lead to the same results as obtained in Ref. 7, thus establishing the correctness of the proposed S matrix to order $1/N$.

V. CONCLUSIONS

In the present paper we have developed a method which allows not only the study of scattering problems of particles with generalized statistics, but also setting up an infrared-regular $1/N$ expansion. We believe this method will also prove useful in other problems involving generalized statistics, for instance the scattering of kinks in the usual Gross-Neveu model.[17] From an esthetic point of view, the development of a scattering and perturbation theory employing only the field $\hat{\psi}$, without resorting to auxiliary fields, would be highly desirable and is now under investigation.

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